A rank-dependent utility model of uncertain lifetime, time consistency and life insurance

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Abstract

In a continuous time life cycle model of consumption with uncertain lifetime and no "pure time preference", we use a non-parametric specification of rank dependent utility theory to characterize the preferences of the agents. From a normative point of view, the paper discusses the implication of adding an axiom of time consistency to the former model. We prove that time consistency holds for a much wider class of probability weighting functions than the identity one characterizing the expected utility model. This special class of probability weighting functions provides foundations for a constant subjective rate of discount which interact multiplicatively with the instantaneous conditional probability of dying. We show that even if agent are time consistent, life annuities no more provide perfect insurance against the risk to live.

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Key words : intertemporal choice; life cycle theory of consumption and saving; uncertain lifetime; life insurance; time consistency; rank dependent utility.

Thanks:

As part of the behavioral revolution, the traditional exponential discounting model has been at stack in the last thirty years. Following the pioneering work of Ainslie (1975) and Thaler (1981), many "anomalies in intertemporal choice" (Loewenstein and Prelec, 1992) have been documented (see Frederick et al., 2002, for a survey).

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There is a long tradition in economics to distinguish two kinds of primitive for explaining discounting. First, discounting can be explained by purely psychological factors, such as impatience, captured by the discount function. If the discount function is exponential as in the seminal model proposed by Samuelson (1937), then time preference is characterized by a "pure rate of time preference" (*i.e.* the log derivative of the discount function) that is invariant with time and the level of consumption. Even if some authors have considered early the possibility for the discount factor to be "non-exponetial" (for example Yaari, 1964; Harvey, 1986), it is only with the behavioral revolution that alternative *ad hoc* parametrical discount functions have been proposed and used systematically in the applied economics literature. Among them the "quasi hyperbolic" discount function (Phelps and Pollak, 1968; Laibson, 1997) is probably the most popular.

The second explanation for discounting is just to consider that future prospect are uncertain. In this case, it is reasonable to consider that the utility of future prospects will be weighted according to their probability to be effectively consumed at the given date (see Sozou, 1998; Dasgupta and Maskin, 2005, for a general discussion of that topic). Among this literature, models of intertemporal choice with uncertain lifetime, pioneered by Yaari (1965), are good tools to investigate the theory of discounting. Yaari (1965)'s seminal paper was considering expected utility maximizers with known probability distributions of the "age of death", and standard exponetial discounting life cycle-utility. More recent models have considered various kind of more sophisticated utility framework to deal with lifetime uncertainty. For example, Moresi (1999) is considering an application of Selden (1978)'s "ordinal certainty equivalent hypothesis". Bommier (2006) considers a concave transformation of the lifecycle utility to explain "risk aversion with respect to length of live". Halevy (2008) uses Yaari (1987)'s "Dual theory of choice". Ludwig and Zimper (2012) and Groneck et al. (2012) consider a non-bayesian treatments of ambiguous survival probability.

The imbrication of risk and time preference is also of first importance for analyzing life insurance, which was Yaari (1965)'s initial purpose. In this paper, we build a model of intertemporal choice of consumption and saving with uncertain lifetime in which the agent psychologically transforms her survival probability distribution, like in Quiggin (1982)'s *rank-dependent utility model*, or in Tversky and Kahneman (1992), cumulative prospect theory. The idea of introducing rank dependent utility in this setting has been already explored by Drouhin (2001) and Bleichrodt and Eeckhoudt (2006). The originality of this paper is that we use continuous time modeling[1] and

[1] Like in Yaari (1965).

optimal control to solve the model. With that methodology, we are able to discuss the important topic of time consistency, the main criteria of rationality over time. Following Strotz (1956), there exists a conventional wisdom in considering that any departure from exponentially discounted expected utility will imply time inconsistency. This paper will give a counterexample. In particular, it shows that any agent who transforms the probability distributions of the age of death with a power function are time consistent. That provides foundations for a coefficient of time preference that interact multiplicatively with the probability of dying, instead of additively in Yaari's approach. However, it also demonstrates that life annuities do not necessarily provide perfect insurance, even in the case in which agents are time consistent[2].

The plan of the paper is as follows. Section 1 presents the utility functional used. Section 2 solves the model in absence of life annuities. Section 3 discusses time consistency. Section 4 solves the model when agent has access to life annuities. Finally section 5 concludes.

1 A rank dependent utility model of consumption and savings with uncertain lifetime

We consider an agent's choice of her consumption profile. A consumption profile is a function of time defined on the interval $[0, T]$, with 0 the age of birth and T an arbitrary constant, interpreted as the maximum possible life duration for the agent. Because we are interested in understanding the way the timing of decision influences the choice of the consumption profile, we will denote by $t \in [0, T)$, the date of the decision.
Let us consider, in a first step, the case in which the agent, alive at date t, knows for sure her age of death s.
H1 If an agent knows with certainty her date of death s, her intertemporal preferences can be represented by an intertemporal utility functional assumed to be additive, and stationary, with no "pure time preference":

$$V_t(c, s) = \int_t^s u(c(\tau))\, d\tau \tag{1}$$

with $u'(c(\tau)) > 0$ and $u''(c(\tau)) < 0$.
H2 (monotonicity according to lifespan). $\forall c : \quad s' > s \Rightarrow V_t(c, s') > V_t(c, s)$
H1 and H2 implies that u is positive. H2 means that for a given consumption profile, outcomes will be always ranked according to lifespan. When

[2]The only exception being the case of expected utility.

introducing uncertainty our model will be a natural candidate for using rank dependent utility. The fact that we do not postulate any kind of "pure" time preference implies that, in our model, lifetime uncertainty is the only primitive of time preference.

The agent actually does not know with certainty her age of death. We assume that for a living agent, at each age t, the age of death $s \geqslant t$ is an absolutely continuous random variable defined on the interval $[t, T]$. We denote by $\pi_t(s) > 0$ the probability density function of the random variable, assumed to be differentiable at least once and $\Pi_t(s)$ the cumulative distribution function. We thus have:

$$\Pi_t(s) = \int_t^s \pi_t(\tau)\, d\tau$$

with $\Pi_t(T) = 1$.
$\Pi_t(s)$ can be interpreted as "the probability of being dead at age s, knowing you are alive at date t", and, $(1 - \Pi_t(s))$ "the probability of being alive at date s, knowing you are alive at date t". We can derive from Bayes formula that for $s \geqslant t' \geqslant t$:

$$(1 - \Pi_{t'}(s))(1 - \Pi_t(t')) = (1 - \Pi_t(s)) \tag{2}$$

and

$$\pi_{t'}(s) = \frac{\pi_t(s)}{1 - \Pi_t(t')} \tag{3}$$

In the special case where $s = t'$, we get the *hazard rate* at date s:

$$\pi_s(s) = \frac{\pi_t(s)}{1 - \Pi_t(s)} \tag{4}$$

We are thus facing a special problem of choice in uncertainty. If we assume that the agent is an expected utility maximizer as in Yaari (1965), we have:

$$EV_t(c) = \int_t^T \int_t^s u(c(\tau) d\tau d\Pi_t(s) \tag{5}$$

If we now retain a more general model of choice under uncertainty, in which agents transform outcomes and probability distributions as in Quiggin (1982, 1993) or Tversky and Kahneman (1992), we have :

$$RDU_t(c) = \int_t^T \int_t^s u(c(\tau) d\tau dh(\Pi_t(s)) \tag{6}$$

with (h) a probability weighting function assumed to be continuous an twice differentiable, and such that: $h(0) = 0$, $h(1) = 1$ and $h'(\Pi_t(s)) \geqslant 0$.

Let us notice that (5) is a special case of (6) when $h(\Pi_t(s)) = \Pi_t(s)$. Integrating (6) by parts, we obtain:

$$RDU_t(c) = \int_t^T (1 - h(\Pi_t(s)))\, u(c(s))\, ds \tag{7}$$

For the agent, the expected present value at date t of the utility stream between t and T is the integral over this interval of the product of the utility of consumption at each date s of the interval with the subjective weight given by the agent to the event "being alive at date this date s". Equation (11) makes explicit our initial intuition within the continuous time framework. The factor $f_t(s) = (1 - h(\Pi_t(s)))$ is the *discount factor* applied to utility of the consumption at date s viewed from date t. It depends only on the probability distribution of the ages of death and the subjective transformation of this probability distribution. It is continuous, derivable and strictly decreasing from one to zero on the interval $[t, T]$.
Taking the log-derivative of this discount factor, we can also define the *rate of discount* of utility at date s viewed from date t:

$$\theta_t(s) \overset{\text{def}}{=} \frac{h'(\Pi_t(s)))\, \pi_t(s)}{1 - h(\Pi_t(s))} \tag{8}$$

Thus we can rewrite the intertemporal rank-dependent utility functional (7):

$$RDU_t(c) = \int_t^T \exp\left(\int_s^t \theta_t(\tau)\, d\tau\right) u(c(s))\, ds \tag{9}$$

At this stage, we just want to notice that the mathematical structures of those discount factor and rate provides a very interesting case. On the one hand, this mathematical structure is much more general than the one that prevails in traditional exponential or hyperbolic model of intertemporal choice. On the other hand, the mathematical structure is also much more precise than the most general form studied by Yaari (1964) where the discount factor is only assumed to be positive and differentiable.

As in Drouhin (2001), the *rate of discount* can be decomposed in two factors. The first one, $\pi_t(s)$, the probability density associated with the event "dying at date s, knowing that your alive at date t", can be interpreted as the *objective part of time preference.* The second one, $h'(\Pi_t(s))/(1 - h(\Pi_t(s)))$, depends on the way the agent transforms probability distributions. It can be interpreted as the *subjective part of time preference.* The *objective part* is exactly the same as in Yaari (1965). The *subjective part* stems from the rank-dependent formulation. It will be of first importance when we will discuss the consequences of this formulation for standard results on time consistency and life insurance.

We are now going to investigate the properties of the choice of the optimal consumption path made by an agent at date t.

2 The optimal consumption path with no life annuities

To express the optimal consumption path, we have first to define the feasible set of consumption profiles. We assume that at each date s the living agent receives a flow of non-financial income $w(s)$ assumed to be continuous and differentiable and a flow of financial income proportional to her asset $(a\,(s))$. Those incomes are either used for current consumption or saved for future consumption. In this part we assume that there are no life annuities. The only asset available for savings is standard bond earning a constant rate of interest r. Thus, at each time s, the standard intertemporal budgetary constraint holds:

$$\forall s \in [0, T]\,, \qquad \dot{a}(s) = w(s) + ra(s) - c(s) \tag{10}$$

We also assume that there is no "bequest motive" implying that an agent living its maximum possible life-duration will choose to leave no bequest $a(T) = 0$. Thus, if between t and T, we sum the differential constraints (10) at each date weighted by the economical discount factor $\exp(-r\,s)$, we obtain after some simple manipulations that, for an agent living the maximum possible life-duration :

$$a(t) + \int_t^T w(s)e^{-r(s-t)}ds = \int_t^T c(s)e^{-r(s-t)}ds \tag{11}$$

This is the very standard life cycle budgetary constraints, the present value of all incomes over the life cycle is equal to the present value of the consumption stream.

Thus we can express the total stock of assets a date t:

$$a(t) = \int_0^s (w(s) - c(s))e^{-r(s-t)}ds \tag{12}$$

We now consider an agent at date t who has to decide her optimal consumption path between t and T. We denote $c_t(s)$ the optimal consumption path decided at date t for the time interval $[t, T]$. Thus $c_t(s)$ is the solution of the following program:

$$\mathcal{P}_t \quad \begin{cases} \max\limits_{c} RDU(c) = \displaystyle\int_t^T (1 - h(\Pi_t(s)))\, u(c(s))\, ds \\ \text{u.c.} \qquad \dot{a}(s) = w(s) + r\ a(s) - c(s)) \\ \qquad\qquad a(t) = cst \\ \qquad\qquad a(T) = 0 \end{cases}$$

Because of the continuity of (w) (h) and (Π_t) and the continuity and strict concavity of (u) this program can be shown to admit a unique solution that will be continuous and derivable[3]. Applying *Pontryagin's maximum principle*, the resolution of such problem implies to solve a system of differential equations. If, for not losing generality of the results, we refuse to specify special "easy to use" functional form for utility, earnings and probability distribution of the age of death, the only thing we can do is to derive the rate of growth of the optimal consumption path planned at date t.

Proposition 1. *Without life annuities, at each date s, the rate of growth of the optimal consumption path planned at date t is:*

$$\frac{\dot{c}_t(s)}{c_t(s)} = \frac{r - \theta_t(s)}{\gamma_t(s)} \tag{13}$$

$$\textit{with} \quad \gamma_t(s) \overset{def}{=} -\frac{u''(c_t(s))}{u'(c_t(s))} c_t(s) \tag{14}$$

the coefficient of relative resistance toward intertemporal substitution

Proof: The Hamiltonian of agent's program is:

$$H = (1 - h(\Pi_t(s)))\ u(c(s)) + \lambda(s)(w(s) + r\, a(s) - c_t(s))$$

First order conditions gives:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow \lambda(s) = (1 - h(\Pi_t(s)))\, u'(c_t(s)) \tag{15}$$

$$\frac{\partial H}{\partial a} = -\frac{d\lambda}{ds}(s) = -\dot{\lambda}_s \Rightarrow \frac{\dot{\lambda}_s}{\lambda_s} = -r \tag{16}$$

[3]For a purpose of simplicity, we have not taken into account the borrowing constraints that is usually associated with the model with no life annuities/insurance. See Leung (1994) for an extensive discussion of this topic.

Taking the logarithm of (15) and differentiating according to s e we get:

$$\frac{\dot{\lambda}_s}{\lambda_s} = -\frac{h'(\Pi_t(s))\,\pi_t(s)}{1-h(\Pi_t(s))} + \frac{u''(c_t(s))}{u'(c_t(s))}\frac{dc}{ds}(s) \tag{17}$$

Comparing (16) and (17), and using definition (8), we deduce proposition 1.□

Proposition 1 is the most general prediction one can make within the life cycle theory of consumption and saving. The rate of growth of the consumption path is the difference between the rate of interest (economic discount rate) and the rate of time preference, both divided by an index of the curvature of the utility function usually referred as the coefficient of relative risk aversion or more properly in this context, according to Gollier (2001), as the resistance to intertemporal substitution. The important point is that, as in Yaari (1965) the rate of time discounting is no more constant and can give a wide variety of possible dynamic for consumption. But contrarily to Yaari (1965) it is not only the properties of the probability distribution of the ages of death that matters. The way agents transform subjectively this probability distribution will also matter. If we want to go further, we have to specify some more restrictions to the model.

3 Time consistency

The intertemporal choice model with uncertain lifetime combine both risk and time. We can specify the model for being consistent with some criteria of rationality. Because it uses rank- dependent utility, our model fulfils necessarily, and by construction, the main axiom of rationality toward risk, *first order stochastic dominance* (Quiggin, 1993). According to decision in time, the main criteria of rationality is *time consistency* proposed by Strotz (1956). What restriction do we have to impose to the probability transformation function to fulfil time consistency? To answer this question we have to define properly the notion of *time consistency.* Using dynamic optimization, we will use the same definition as Strotz (1956); Caputo (2005); Drouhin (2009, 2012). In the absence of new information, an agent is said to be time consistent if she behave in the future as she has planned in the past.

Definition 1 (Time consistency)**.** *If we denote $(c_t)$and (a_t) the solution of the program $\mathcal{P}_t$. If we denote $(c_{t'})$and $(a_{t'})$, the optimal solution of the* *program $\mathcal{P}_{t'}$, with $t \in [t, T]$ and:*

$$\mathcal{P}_{t'} \quad \begin{cases} \max_c RDU(c) = \int_{t'}^{T} \left(1 - h\left(\Pi_t(s)\right)\right) u\left(c(s)\right) ds \\ u.c. \qquad \dot{a}(s) = w(s) + r\, a(s) - c(s)) \\ \qquad\qquad a(t') = a_t(t') \\ \qquad\qquad a(T) = 0 \end{cases}$$

Then an agent is time consistent *if and only if:*

$$\forall t \in [0, T]\,,\ \forall t' \in [t, T]\,, \forall s \in [t', T] :\ c_t\left(s\right) = c_{t'}\left(s\right) \tag{18}$$

A well known corollary of this definition is that for being time consistent the rate of discount at each date s has to be independent from the decision date t.[4] In the special case of expected utility, $\theta_t\left(s\right) = \pi_s(s)$, whatever the form of the probability distribution, it does not depend on the planning decision date, it is thus "time distance independent", time consistency holds. But for other cases the distribution probability of death and the rank-dependent utility give a special mathematical structure to the discount rate and factor. We can notice that in the most general case $\theta_t\left(s\right)$ is time-distance dependent because it depends on $\Pi_t\left(s\right)$. It is a strong presumption for time inconsistency. Is there nevertheless some other cases where time consistency holds?

Proposition 2. *Agent is time consistent if and only if her probability distribution transformation function is of the form $h\left(x\right) = 1 - \left(1 - x\right)^{\alpha}$ with $(\alpha > 0)$.*

Proof:
(sufficiency)
$h\left(x\right) = 1 - \left(1 - x\right)^{\alpha} \Rightarrow \theta_t\left(s\right) = \alpha\, \pi_s\left(s\right)$
The rate of discount is independent of the planning date so the choice of consumption is time consistent.
(necessity)
If the agent is time consistent, she fulfills equation (18). As $c_t(s)$ is strictly positive and differentiable, it implies that:

$$\forall t \in [0, T]\,,\ \forall t' \in [t, T]\,, \forall s \in [t', T] :\ \frac{\dot{c}_t(s)}{c_t(s)} = \frac{\dot{c}_{t'} t(s)}{c_{t'}(s)}$$

Taking into account(13), (8), and (3) it implies that:

[4] See, for example, Drouhin (2012) for a rigorous proof.

$\forall t \in [0,T]\,,\ \forall t' \in [t,T]\,, \forall s \in [t',T]:$

$$\frac{h'\left(\Pi_t\left(s\right)\right)}{1-h\left(\Pi_t\left(s\right)\right)} = \frac{h'\left(\Pi_{t'}\left(s\right)\right)}{\left(1-h\left(\Pi_{t'}\left(s\right)\right)\right)\left(1-\Pi_t\left(t'\right)\right)} \tag{19}$$

This equation should hold in the particular case where $s = t'$. Considering this case and remarking that $\Pi_t\left(t\right) = 0$, equation (19) also implies :

$$\forall t \in [0,T]\,,\ \forall t' \in [t,T]: \frac{h'\left(\Pi_t\left(t'\right)\right)}{1-h\left(\Pi_t\left(t'\right)\right)} = \frac{h'\left(0\right)}{\left(1-\Pi_t\left(t'\right)\right)} \tag{20}$$

This is a first order differential equation with a set of solutions fully described by $h\left(x\right) = 1-\left(1-x\right)^{\alpha}$, with $\alpha = h'\left(0\right)$. $\square$

The distribution of the probability of dying and its treatment within Rank-Dependent Utility Theory of choice added with an axiom of time consistency gives behavioral foundations to a model of intertemporal choice that is rather simple and not less intuitive than the standard discounted utility model. Some points should be noticed.

1. The expected utility model is not the only one compatible with time consistency. The rank-dependent utility model with a power function for transforming the probability distribution[5] implies also time consistency. This gives behavioral foundations for a model of intertemporal choice that is different from the original discounted expected utility model.

2. When agent is time consistent we have:

$$\theta_t(s) = \alpha\, \pi_s(s) \tag{21}$$

   The utility functional can be rewritten :

$$RDU_t\left(c\right) = \int_t^T e^{\alpha \int_s^t \pi_\tau(\tau)d\tau} u\left(c\left(s\right)\right)ds \tag{22}$$

3. The parameter α can be interpreted as *multiplicative factor of time preference*. If $\alpha > 1$, this means that the agent gives a psychological weight to present consumption more important than the instantaneous probability of dying. In this case the agent will demonstrate *preference for present consumption*. In a RDU/ Cumulative prospect theory, the behavior is interpreted as "pessimistic" in the sense that she tends to

[5]Diecidue et al. (2009) provide axiomatic foundations for such probability transformation function.

overweight her probability of dying[6]. In the opposite, if $\alpha < 1$, the agent will demonstrate a kind of *preference for future consumption*, she underweight her probability of dying ("optimistic" behaviour).

4. InYaari (1965) the rate of discount was:

$$\theta_t(s) = \theta + \pi_s(s) \tag{23}$$

It means that our model has the same level of complexity as Yaari's model. It depends only on one parameter and on the conditional probability density of the event "dying at date s knowing you are alive at date s. In Yaari (1965), the parameter θ interact additively with the probability of dying. In our's the parameter α interact multiplicatively. Both parameters can be interpreted as measuring time preference. But a huge difference is that in Yaari (1965) discounted expected utility model, as in standard economic theory, the parameter θ is postulated as a coefficient of pure time preference, in a rather *ad hoc* manner. In the contrary, in our time consistent rank dependent utility model, we have not postulated any ad-hoc form of time preference. The coefficient α just stems from the axiom of time consistency, as we have demonstrated[7].

To comment further, we have to consider special distributions of the age of death. The most simple case (and not so unrealistic) is the one of a constant hazard rate (standard Poisson process). For that to be possible we have to allow the maximum duration of life, T, to go to infinity. Let us assume that, for all $s \in \mathbb{R}^+$, $\pi_s(s) = \pi = cst$. In this case the intertemporal utility functional is equivalent to the exponential discounting model (with $\alpha\,\pi = \theta$):

$$RDU_t\,(c) = \int_t^{+\infty} e^{-\alpha\,\pi\,(s-t)} u\,(c\,(s))\,ds \tag{24}$$

The standard model of exponential discounting is just a simplified version of our general rank-dependent utility model of intertemporal choice with uncertain lifetime in the special case of time consistency and constant hazard rate.

If we adopt a more realistic model of uncertain lifetime, like for example the Gompertz law, then the hazard rate is increasing with age. This has an important consequence :

[6] See, for example, Wakker (2010), 172-176, for an extensive discussion of probability transformation as pesimism/optimism

[7] Same degree of complexity, less ad-hocity, according to Occam's razor principle, our model is a better candidate for modeling uncertain lifetime and time insurance!

Proposition 3. *If the consumption stream is bounded and if there exists an age $\hat{t}$ such that for all $t > \hat{t}$,* $\frac{\partial \pi_t(t)}{\partial t} > 0$*, then for all* $T \in \mathbb{R}^+ + \{+\infty\}$ *then the intertemporal rank dependent utility functional* (22) *is always definite.*

Proof: Obvious. □

This proposition means that our concept of multiplicative time preference is more robust than the usual rate of pure time preference of the exponential discounting model. In particular preference for present consumption in our model ($\alpha > 1$) is not a prerequisite for our utility functional to be definite event when the horizon is infinite. From a behavioral perspective it means that our model is a tool to explore possibilities than cannot be addressed with the standard discounted expected utility model.

4 The optimal consumption path with life annuities

As in Yaari (1965), we will now assume that agents have access to actuarial notes issued by insurance companies or pension funds. Those notes are contingent asset that pay $R(t)$ as long as the agent is alive, and 0 after her death. If insurance companies refund themselves on the bond market at the rate r, and if those notes are actuarially fair, then it is well known that:

$$R(t) = r + \pi_t(t) \tag{25}$$

When there is no bequest motives, standard bonds are strictly dominated by life annuities. Thus differential constraint of the program can be rewritten:

$$\forall s \in [0, T], \qquad \dot{a}(s) = w(s) + R(s)a(s) - c(s) \tag{26}$$

We can now deduce the property of the optimal intertemporal consumption profile when the agent have access to life annuities.

Proposition 4. *When the agent has access to life annuities, then at each date s, the rate of growth of the optimal consumption path planned at date t is:*

$$\begin{aligned} \frac{\dot{c}_t(s)}{c_t(s)} &= \frac{R(s) - \theta_t(s)}{\gamma_t(s)} \\ &= \frac{r + \pi_s(s) - \frac{h'(\Pi_t(s))\pi_t(s)}{1-h(\Pi_t(s))}}{\gamma_t(s)} \end{aligned} \tag{27}$$

Proof: We proceed exactly the same way as in Proposition 1. □

The main result of Yaari's model was that when the agent has access to life annuities the rate of growth of the intertemporal consumption profile was no more determined by the conditional probability of dying, and thus was the same as the one of the model with a certain life duration. This is why life annuities are considered as offering *perfect insurance* with the meaning that uncertainty has no more influence on the rate of growth of optimal consumption. Obviously, as Proposition 4 shows, that is generally no more the case in our rank-dependent utility model of intertemporal choice.

Corollary 4-a. *If the agent is time consistent and has access to life annuities, the rate of growth of the optimal consumption path planned at date t is:*

$$\frac{\dot{c}_t(s)}{c_t(s)} = \frac{r + (1-\alpha)\pi_s(s)}{\gamma_t(s)} \tag{28}$$

This result is important. In the case $\alpha = 1$ (expected utility), we retrieve Yaari's result. But if $\alpha \neq 1$, then we have not perfect insurance, *i.e.* the conditional probability of dying still determine the rate of growth of the consumption profile. What is is important to notice is that, in this last case, the agent is fully rational, she fulfils simultaneously first order stochastic dominance and time constancy. Nevertheless, there is no more perfect insurance in this case[8].

This result is important for analyzing social security. Within Yaari's model fully funded social security is considered as equivalent with life annuities, and thus provides perfect insurance. In our model this is no more necessary true, even for *fully rational* agents.

5 Conclusion

Conventional wisdom generally considers exponential discounting and expected utility as being the only models of choice compatible with full rationality. If those models do not fit the actual behavior of agents, then it is legitimate to consider alternative descriptive/behavioral models. If you believe in the conventional wisdom, you will deduce that agents are no more rational. Obviously, from a normative point of view, that has very important implications for policy design. However, this paper proves, in the case of life

[8]Using Selden (1978)'s *ordinal certainty equivalent* instead of *rank dependent utility*, Moresi (1999) arrives to a very similar conclusion, in the special case of an iso-elastic per period utility function. RDU is more general because it implies no restriction on the per-period utility function.

cycle model of choice with uncertain lifetime, that the conventional wisdom may be false. It means that, as suggested by Zeckhauser and Viscusi (2008), we have to better distinguish what is behavioral and what is normative.

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